**GRADIENT ELASTICITY FORMULATION**

The simplest form of gradient elasticity theory proposed by Aifantis the classical stress-strain relation is augmented by a second-order strain gradient as

 (1a)

*  and  are stress and strain component vectors
* **D** is the elastic modulus matrix
*  is the length scale parameter

The above equation can be rewritten as

 (1b)

where  (2)

Eq. (2) is called as the *explicit* form that shows the relation between the nonstandard (nonlocal/averages trains) with the local strain . By employing a series approximation of an integral expression and truncating the high order terms, the *implicit* relation between and  is given as:

 (3)

**SPATIAL DISCRETIATION WITH EXPLICIT FORMUALTION**

The equilibrium equation

 (4)

Or in matrix form

 (5)

Where the differential operator **L**



The weak form of the equilibrium equations:

 (6)

Where  is the virtual displacement that correspond to the stress field 

From Eq. (6), take the integrate by part with respect to the stress term, which yields

*  (7)
*  (8)
*  (9)

Substituting Eq. (1a) into Eq. (9), we have

*  (10)
*  (11)
*  (12)
*  (13)
*  (14)

It is assumed that the derivatives of vanish on the boundary, hence the last boundary integral in above equation cancels.

Discretizing the test and the trial functions via and where **N** is the matrix of shape functions

 (15)

where a summation over  is implied and with **B = LN**, the above equation leads to

 (16)

Or in matrix form

 (17)

where the standard stiffness matrix given by

 (18)

The high-order stiffness matrix is given by

 (19)

And the external force vector is given by

 (20)

In Eq. (16), it can be seen that the high-order stiffness matrix require second order derivative of the stiffness functions and the shape function need to be C1 continuous; therefore the employment of NURBS basic functions in this situation seems to be reasonable and promising.

For the discretization of the implicit formulation, Askes and Aifantis [1] demonstrated this formulation become manifest in implementation compared to the explicit one; for instance, the occurrence of the zero block destroys the positive definiteness of the system, the number of unknowns increases significantly in comparison with explicit formulation…

**Reference**

[1] H. Askes and E. C. Aifantis, “Numerical modeling of size effects with gradient elasticity - Formulation, meshless discretization and examples,” *Int. J. Fract.*, vol. 117, no. 4, pp. 347–358, Oct. 2002.